АГ 29.10.12

$(a-b)\times(a+b) = 2a\times b$

$\underbrace{a\times a}_{0} + a\times b - b\times a - \underbrace{b\times b}_{0}$

"0 - заради синусите = 0"

$a\times b - b\times a = a\times b + a\times b = 2a\times b$

$|a|=2$, $|b|=3$, $(ab)_e = \frac{\pi}{2}$

$|a||b|\sin(ab)_e = 2\cdot 3\cdot \sin\frac{\pi}{2} = 2\cdot 3\cdot 1 = 6$

---

a, b, c - ЛНЗ

$a\times b$, $b\times c$, $c\times a$ - ЛНЗ

$\alpha(a\times b) + \beta(b\times c) + \gamma(c\times a) = 0$

умножаваме по abc $\Rightarrow$ $a\perp b\perp c\perp a$ и $\alpha abc = 0$

$a\times b = c\times d$ / $a-d, b-c$ ЛЗ

$a\times c = b\times d$

?! $a = \frac{c\times d}{b}$, $a = \frac{b\times d}{c}$ разглеждаме векторно

Не се използва

$(a-d)\times(b-c) = a\times b - a\times c - d\times b + d\times c \Rightarrow$

но $a\times b = c\times d$ и $a\times c = b\times d$ - следзамества не $\Rightarrow$

$c\times d - b\times d - d\times b + d\times c = 0$

$A(1,2,-1)$; $B(3,1,2)$ $\quad \vec{AB} = ?$; $|AB| = ?$

$\vec{AB} = (3-1, 1-2, 2-(-1)) \Rightarrow \vec{AB}(2,-1,3)$

$|AB| = |\vec{AB}| = \sqrt{2^2+(-1)^2+3^2} = \sqrt{14}$

$a \times b = b \times c = c \times a \neq \vec{0}$

Да се докаже: $a + b + c = \vec{0}$

$a \times b = b \times c$

$a \times b - b \times c = \vec{0}$

$a \times b + c \times b = \vec{0}$ → представяне b ⇒

$(a+c) \times b = \vec{0} \Rightarrow b = \lambda(a+c)$ или $b = -a - c$ т.е

$a + b + c = \vec{0}$

$b \times c = \lambda(a+c) \times c$

$b \times c = \lambda a \times c + \lambda \underbrace{c \times c}_{=0} \Rightarrow b \times c = \lambda a \times c$

но $b \times c = c \times a \Rightarrow c \times a = \lambda a \times c \Rightarrow \lambda = -1$

$a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = \vec{0}$

(тъждество на Якоби) unchain my heart

Формули

$$a \times (b \times c) = (ac)b - (ab)c$$
$$(a \times b) \times c = (ac)b - (bc)a$$

$(ac)b - (ab)c + (ba)c - (bc)a + (cb)a - (ca)b = \vec{0}$

ако a и c са $\mathbb{R}3$ имаме асоциативност (колинеарни)

$a = (2, 1, 0)$; $b = (-6, 12, 1)$ / търси се: $|a|, |b|, ab, (ab)_c, a \times b$

$|a| = \sqrt{2^2 + 1^2} = \sqrt{5}$; $|b| = \sqrt{(-6)^2 + 12^2 + 1^2} = \sqrt{181}$

$ab = 2.(-6) + 1.12 + 0.1 = 0$

$$\cos(ab)_c = \frac{ab}{|a||b|} = \frac{0}{\sqrt{5}\sqrt{181}} = 0 \Rightarrow (ab)_c = \frac{\pi}{2}$$

(2)

$$a \times b = \left(\begin{vmatrix} 1 & 0 \\ 12 & 1 \end{vmatrix} \begin{vmatrix} 0 & 2 \\ 1 & -6 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ -6 & 12 \end{vmatrix} \right) = (1, -2, 30)$$